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Stock reduction analysis using catch-at-length data: Length-SRA

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A R T I C L E I N F O

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A B S T R A C T

Many modern stock assessments estimate age- or length-based selectivity, often using simple parametric functions describing asymptotic or dome-shaped selectivity. We present a length-based stock reduction analysis (Length-SRA), which bypasses the requirement of estimating selectivity by calculating exploitation rate at length directly from observed catch-at-length data. We test the performance of Length-SRA using a simulation–evaluation framework under three exploitation rate trajectories and under fixed and time-varying selectivity scenarios. We also explore the impacts of misspecification of growth parameters. The Length-SRA yields low bias in parameter estimates and management benchmarks and is relatively accurate when tracking changes in selectivity through time. We use Length-SRA to assess two species, Pacific hake and Peruvian jack mackerel, showing that selectivity is quite variable in both species over time, leading to time-varying management reference points. Length-SRA provides assessment results with accuracy comparable to other methods, such as Virtual Population Analysis and Statistical Catch at Age Analysis, with the additional advantage of providing estimates of selectivity over time.

1. Introduction

Modern stock assessment typically attempt to fit population dynamics models to catch-at-age and at-length data, in hopes of extracting information from these data about age/size selectivity, cohort strength and fishing mortality patterns (Hilborn and Walters, 1992; Method and Wetzel, 2013). Some assessment methods attempt to ignore the length frequency data, by converting these data to age compositions using age-from-length tables, perhaps using iterative methods to estimate proportions of fish at age for each length interval (Kimura and Chikuni, 1987). In cases where age data are lacking, methods such as MULTIFAN-CL (Fournier et al., 1998) attempt to obtain estimates of selectivity, fishing mortality and population dynamics parameters using only size composition data. Combined with a few assumptions regarding the structure and variability in length-at-age, this procedure can even be used to attempt to recover information about changes in body growth patterns if there is a strong age-class signal in the length composition data (Fournier et al., 1998). It is typical for results from length-based assessment models to show substantial deviations between predicted and observed length distribution of catches, reflecting both sampling variation in the length composition data and incorrect assumptions about the shapes and stability of growth and selectivity patterns (Hilborn and Walters, 1992).

When dealing with length-based stock assessments, it is not uncommon to encounter conflict between data types. For example, there may be a conflict between an index of abundance and length-composition data (Punt et al., 2013). For many length-based assessment methods, the poor fit to the length composition data arise from the inability of the models to track changes in fisheries selectivity over time (Gulland and Rosenberg, 1992; Punt et al., 2013). Changes in fisheries selectivity over time are thought to occur for many fisheries, and if not taken into account can lead to biased assessment results. However, many data-poor assessment methods have to rely on constant selectivity assumptions (Gulland and Rosenberg, 1992; Rudd and Thorson, 2017).

Selectivity to fishing is the combination of two processes: vulnerability to the fishing gear and availability of the fished population in the area being fished (Beverton and Holt, 1957). Both processes can vary over time and therefore modify the resulting selectivity. Although vulnerability process can often be directly measured using gear experiments, availability is generally harder to measure as it depends on the size-based distribution of the exploited population and the spatial distribution of the fishing fleet. Fish movement, size-structured changes in fish distribution, and changes in fleet distribution, can all affect availability and consequently lead to changes in selectivity. Such

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changes are not uncommon (Sampson and Scott, 2012), but are usually difficult to track over time. This difficulty is associated with an inability to distinguish between changes in fishing mortality and changes in selectivity in most age- and length-based stock assessment methods. For this reason, many assessment methods rely on ad hoc parametric selectivity models that may or may not include changes over time (Maunder et al., 2014). If misspecified, such models might lead to severe bias in estimates of fishing mortality and other parameters, which could result in misleading management advice (Martell and Stewart, 2014).

We outline an alternative approach to assessment modeling that begins by assuming that the assessment model should exactly reproduce the observed catch length-composition. This approach follows the dynamics of an age-structured stock reduction analysis (SRA) (Kimura et al., 1984; Kimura and Tagart, 1982; Walters et al., 2006), which follows a "conditioned on catch" format, in which the catch length-composition is assumed to be known without error. The observed catches-at-age are then subtracted from modeled numbers at age to project numbers at age over time. A good review of SRA-type models is provided in Thorson and Cope (2015). The assumption of known catch composition is analogous to the classical assumption in virtual population analysis that reconstructed numbers at age should exactly match observed catch-at-age data (Hilborn and Walters, 1992). The suggested approach may have two key advantages over statistical catch-at-age and/or catch-at-length methods: (1) it does not require estimation of age- or size-selectivity schedules, and (2) catch-at-length data are commonly available for every year, even when age-composition sampling has not been conducted.

We named this approach a Length-SRA assessment model. We present the model formulation, demonstrate its performance using a simulation–evaluation analysis and apply it to actual data from the Peruvian jack mackerel (*Trachurus murphyi*) and Pacific hake (*Merluccius productus*) fisheries.

2. Methods

2.1. Stock reduction analysis with catch-at-length data – length-SRA

Length-SRA proceeds through the following steps: (1) compute numbers-at-age (based on recruitment estimates and mortality in the previous year); (2) convert numbers-at-age into numbers-at-length using the proportions of individuals at length given each age class; (3) calculate the exploitation rate-at-length using numbers-at-length and observed catch-at-length; (4) convert the exploitation rate-at-length to exploitation rate-at-age; and (5) compute numbers in the following year using the exploitation rate at age, natural mortality, and recruitment estimates.

The model requires data on the length composition of catch in numbers (used in step 3), a prior distribution for the recruitment compensation ratio, and a survey index of abundance that is used to tune the model parameters to the most likely stock abundance trajectory. The model also requires good estimates of growth parameters, variability around mean length-at-age, and natural mortality. The stock assessment and simulation routines were written in ADMB (Fournier et al., 2012) and are available on github.com/catarinawor/length_SRA. The notation used in the stock assessment and simulation routine is defined in Tables 1 and 2).

A crucial component of the Length-SRA is the calculation of proportions of individuals at length given each age class ($P_{l|a}$ – Eqs. T3.1–T3.5). The calculation of such proportions (Eq. T3.1) relies on four main assumptions regarding the distribution of length-at-age: (1) mean length-at-age follows a von Bertalanffy growth curve (Eq. T3.4), (2) length-at-age is normally distributed (Eqs. T3.1–T3.3), (3) the standard deviation of the length-at-age is known (e.g. eq. T3.5), and (4) $P_{l|a}$ is constant for all lengths equal or greater than a maximum length L and lower than a minimum length l_o(Eq. T3.1).

Table 1
Indexes, variable definition, and values used in simulation–evaluation.

Symbol	Value	Description
l	$\{l_o, \ldots, L\}$	Central point of length bin, $L = 72$ cm
a	$\{a_o, \ldots, A\}$	Age-class, $A = 20$ years
t	$\{1, \ldots, T\}$	Annual time step, $T = 50$ years
a_o	1	First age or age of recruitment
l_{bin}	2 cm	Size of length bin
l_o	26 cm	Central point of first length bin
$init$	21	Annual time step in which data starts to be reported
Distribution of length given age		
L_∞	68 cm	Maximum average length
K	0.3	Rate of approach to L_∞
t_o	−0.1	Theoretical time in which length of individuals is zero
cv_l	0.08	Coefficient of variation for length-at-age curve
$P_{l\|a}$		Matrix of proportions of length-at-age
Φ		Standard normal distribution
$zl_{a,l}$		Normalized z score for lower limit length bins
$zu_{a,l}$		Normalized z score for upper limit length bins
bl_l		Lower limit of length bins
bu_l		Upper limit of length bins
$\bar{L}_a$		Mean length-at-age
σ_L		Standard deviation of length-at-age
Population dynamics		
R_o	100	Average unfished recruitment
κ	10	Goodyear recruitment compensation ratio
S	0.7	Natural annual survival
σ_R	0.6	standard deviation for recruitment deviations
w_t	$\mathcal{N}(0, \sigma_R)$	Recruitment deviations for years {init-A-a_o,…,T}
$N_{a,t}$		Numbers of fish at age and time
SB_t		Spawning biomass at time
mat_a		Proportion of mature individuals at age
VB_t		Biomass that is vulnerable to the survey at time t
v_a	{0,0.5,1, …,1}	Survey vulnerability at age
$U_{a,t}$		Exploitation rate at age and time
$U_{l,t}$		Exploitation rate at length and time
$C_{l,t}$		Catch at length and time
$N_{l,t}$		Numbers at length and time
lx_a		Unfished survivorship at age
ϕ_e		Unfished average spawning biomass per recruit
$\widehat{\mathrm{sel}}_{l,t}$		Selectivity estimates at length and time

The proportions of length-at-age are used to convert length-based quantities into age-based quantities, which are used to propagate the age-structured population dynamics forward (Table 3). We assume that recruitment follows a Beverton-Holt type recruitment curve (Eq. T3.6), that harvesting occurs over a short, discrete season in each time step (year or shorter), and that natural survival rate is known and constant over time (Eqs. T3.6–T3.10). The computation of numbers-at-age in the initial year (i.e. first year in which data is reported, t_{init}) is different from that in the remaining years (eq. T3.13). Recruitment in the initial year is set to the unfished recruitment level R_o multiplied by random recruitment deviates, which are used to indicate that the population was not at equilibrium at the start of the time series.

We used equilibrium spawner-per-recruit (SPR) quantities to calculate management targets. For illustration purposes we use 40% as a SPR target and use $Yield_{SPR=40\%}$ and $U_{SPR=40\%}$ as target management benchmarks (Table 4 – Eqs. T4.6–T4.14). As in all SPR calculations, the

Table 2
Indexes, variable definition for operating model, MSY quantities, and values used in simulation–evaluation.

Symbol	Value	Description
Operating model		
$sel_{l,t}$		Fishing selectivity at length and time
g, d, k	Vary by scenario	Parameters for selectivity function
U_t	Vary by scenario	Annual maximum exploitation rate
I_t		Index of abundance at time
σ_{I_t}	0.2	standard deviation for index of abundance deviates
q	1.0	Catchability coefficient
τ		Multivariate logistic error term with $\sigma_\tau = 0.1$
Management quantities		
lz_a		Fished survivorship at age
F_z	seq(0.0, 1.0, by = 0.001)	Hypothetical average fishing mortality to calculate management targets
ϕ_z		Average spawning biomass per recruit
ϕ_{eq}		Average exploited biomass per recruit under U_z
$rsel_{a,t}$		Realized selectivity at age and time t
R_{eq}		Average equilibrium recruitment under U_z
$Yield_z$		Equilibrium yield under U_z
$Yield_{target}$		Yield that would reduce spawner per recruit to 40% of unfished levels
U_{target}		Exploitation rate that reduce spawner per recruit to 40% of unfished levels

Table 3
population dynamics for Length-SRA and operating model.

Distribution of length given age

$$P_{l|a} = \begin{cases} \phi(\mathrm{zu}_{a,l}) & l = l_o \\ \phi(\mathrm{zu}_{a,l}) - \phi(\mathrm{zl}_{a,l}) & l < L \\ 1 - \phi(\mathrm{zl}_{a,l}), & l = L \end{cases} \quad \text{(T3.1)}$$

$$\mathrm{zl}_{a,l} = \frac{\mathrm{bl}_l - \overline{L}_a}{\sigma_{L_a}} \quad \text{(T3.2)}$$

$$\mathrm{zu}_{a,l} = \frac{\mathrm{bu}_l - \overline{L}_a}{\sigma_{L_a}} \quad \text{(T3.3)}$$

$$\overline{L}_a = L_\infty \cdot (1 - e^{(-K \cdot (a - t_0))}) \quad \text{(T3.4)}$$

$$\sigma_{L_a} = \overline{L}_a \cdot \mathrm{cv}_l \quad \text{(T3.5)}$$

Population dynamics

$$N_{a,t>\mathrm{init}} = \begin{cases} \dfrac{\frac{\kappa}{\phi_e} \cdot \mathrm{SB}_{t-1}}{1 + (\frac{\kappa - 1}{R_0 \cdot \phi_e}) \cdot \mathrm{SB}_{t-1}} \cdot e^{w_t - \frac{\sigma_R^2}{2}}, & a = a_o \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}), & a_o < a < A \\ N_{a-1,t-1} \cdot S \cdot (1 - U_{a-1,t-1}) + N_{a,t-1} \cdot S \cdot (1 - U_{a,t-1}), & a = A \end{cases} \quad \text{(T3.6)}$$

$$U_{a,t} = \Sigma_l (P_{l|a} \cdot U_{l,t}) \quad \text{(T3.7)}$$

$$U_{l,t} = \frac{C_{l,t}}{N_{l,t}} \quad \text{(T3.8)}$$

$$N_{l,t} = \Sigma_a (P_{l|a} \cdot N_{a,t}) \quad \text{(T3.9)}$$

$$\mathrm{SB}_t = \sum_a (\mathrm{mat}_a \cdot w_a \cdot N_{a,t}) \quad \text{(T3.10)}$$

$$\hat{\mathrm{sel}}_{l,t} = \frac{U_{l,t}}{\overline{U_t}} \quad \text{(T3.11)}$$

$$\mathrm{VB}_t = \sum^a N_{a,t} \cdot w_a \quad \text{(T3.12)}$$

Initial year and incidence functions

$$N_{a,t=\mathrm{init}} = \mathrm{lx}_a \cdot R_o \cdot e^{(w_{t=\mathrm{init}} \cdots w_{t=\mathrm{init}-A+a_o} - \frac{\sigma_R^2}{2})} \quad \text{(T3.13)}$$

$$\phi_e = \sum_a \mathrm{lx}_a \cdot \mathrm{mat}_a \cdot w_a \quad \text{(T3.14)}$$

$$\mathrm{lx}_a = \begin{cases} 1, & a = 1 \\ \mathrm{lx}_{a-1} \cdot S, & 1 < a < A \\ \dfrac{\mathrm{lx}_{a-1} \cdot S}{1 - S}, & a = A \end{cases} \quad \text{(T3.15)}$$

Table 4
Management quantities and operating model.

Operating model

$$N_{a,t=1} = lx_a \cdot R_o \quad \text{(T4.1)}$$

$$U_{l,t} = U_t \cdot \mathrm{sel}_{l,t}^{\mathrm{OM}} \quad \text{(T4.2)}$$

$$C_{l,t} = N_{l,t} \cdot U_{l,t} \cdot P_{l|a} \cdot \tau \quad \text{(T4.3)}$$

$$\mathrm{sel}_{l,t} = \frac{1}{1-g} \cdot \left(\frac{1-g}{g}\right)^g \cdot \frac{e^{d \cdot g \cdot (k-l)}}{1 + e^{d \cdot (k-l)}} \quad \text{(T4.4)}$$

$$I_t = q \cdot \mathrm{VB}_t \cdot e^{(\mathcal{N}(0, \sigma_{I_t}))} \quad \text{(T4.5)}$$

Management quantities

$$\mathrm{lz}_a = \begin{cases} \mathrm{lz}_a = 1 & a = a_o \\ \mathrm{lz}_{a-1} \cdot S \cdot \exp(-F_z \cdot \mathrm{rsel}_{a-1,t}) & a_o < a < A \\ \dfrac{\mathrm{lz}_{a-1} \cdot S \cdot \exp(-F_z \cdot \mathrm{rsel}_{a-1,t})}{1 - S \cdot \exp(-F_z \cdot \mathrm{rsel}_{A,t})} & a = A \end{cases} \quad \text{(T4.6)}$$

$$\mathrm{rsel}_{a,t} = \frac{\frac{U_{a,t-1}}{\overline{U}_{t-1}} + \frac{U_{a,t}}{\overline{U}_t}}{2} \quad \text{(T4.7)}$$

$$\phi_z = \sum_a \mathrm{lz}_a \cdot \mathrm{mat}_a \cdot w_a \quad \text{(T4.8)}$$

$$\mathrm{Target}_\phi = \left|\frac{\phi_z}{\phi_e} - 0.4\right| \quad \text{(T4.9)}$$

$$\phi_{\mathrm{eq}} = \sum_a \mathrm{lz}_a \cdot (1 - \exp(-F_z * \mathrm{rsel}_{a,t})) \cdot w_a \quad \text{(T4.10)}$$

$$R_{\mathrm{eq}} = R_o \cdot \frac{\kappa - \phi_e / \phi_z}{\kappa - 1} \quad \text{(T4.11)}$$

$$Yield_z = R_{eq} \cdot \phi_{eq} \quad \text{(T4.12)}$$

$Yield_{target} = Yield_z$ associated with $min(Target_\phi)$ (T4.13)

$U_{target} = 1 - exp(-F_z)$ associated with $min(Target_\phi)$ (T4.14)

Table 5
Likelihood functions and penalties.

Conditional Likelihood

$$Z_t = log(I_t) - log(VB_t) \quad \text{(T5.1)}$$

$$q = e^{\overline{Z}} \quad \text{(T5.2)}$$

$$\mathrm{Zstat}_t = Z_t - \overline{Z} \quad \text{(T5.3)}$$

$$\mathrm{LL}_1 \sim \mathcal{N}(\mathrm{Zstat} | \mu = 0, \sigma = \sigma_{I_t}) \quad \text{(T5.4)}$$

Penalties

$$P_{wt} \sim \begin{cases} \mathcal{N}(w_t | \mu = 0, \sigma = \sigma_R) & \text{phase} < \text{last phase} \\ \mathcal{N}(w_t | \mu = 0, \sigma = \sigma_R \cdot 2) & \text{phase} = \text{last phase} \end{cases} \quad \text{(T5.5)}$$

Priors

$$\mathrm{prior}(\log(\kappa)) \sim \mathcal{N}(\log(10), \sigma = 0.5) \quad \text{(T5.6)}$$

$$\mathrm{prior}(\log(q)) \sim \mathcal{N}(\log(1.0) \sigma = 0.5) \quad \text{(T5.7)}$$

Objective function

$$Obj = -log(LL_1) + (-log(P_{wt})) + prior(log(\kappa)) + prior(log(q)) \quad \text{(T5.8)}$$

$Yield_{target}$ and U_{target} estimates depend on the selectivity curves calculated for each year (Eq. T4.9) (Table 5).

To assess how well the model tracked changes in selectivity over time, we calculated the resulting selectivity estimates by normalizing the yearly vectors of exploitation rate-at-length ($U_{l,t}$) by the yearly average exploitation rate-at-length ($\overline{U_l}$) (Eq. T3.11), which is more stable than the maximum yearly exploitation rate ($maxU_l$). This happens because observation errors tend to average out over the length classes, diminishing variability of $\overline{U_l}$ in relation to $maxU_l$. When calculating the management targets, we used the same method to calculate the mean selectivity at age (Eq. T4.7). However we also averaged selectivity-at-age over the past two years (Eq. T4.7) to further smooth the curves.

Length-SRA estimates two main parameters: average unexploited recruitment R_0 and the recruitment compensation ratio κ. In addition, the annual recruitment deviations w_t are estimated for all cohorts observed in the model. That is, the number of recruitment deviations is equal to the number of years in the time series plus the number of age classes greater than recruitment age.

The objective function (Eq. T5.8) is composed of a negative log-likelihood component, one penalty, and a prior component for the recruitment compensation ratio κ. The negative log-likelihood component minimizes the differences between the predicted and observed index of abundance (Eq. T5.1). We assume that such differences are lognormally distributed (Eqs. T5.3 and T5.4) and use the conditional maximum likelihood estimator described by Walters and Ludwig (1994) to estimate the survey catchability coefficient q (Eq. T5.2). A lognormal penalty is added to the negative log-likelihood function to constrain annual recruitment residuals so estimates have mean of zero and fixed standard deviation σ_R (Maunder and Deriso, 2003) (Eq. T5.5). This penalty was applied differently in the ADMB parameter estimation phases. The value for the standard deviation for recruitment deviations (σ_R) was set to the input value for most estimation phases. However in the last estimation phase, the value for σ_R was multiplied by 2. This procedure was based on the findings of Schnute and Kronlund (2002) that maximum likelihood variance estimations tends to be lower than real variances. We found that multiplying σ_R by a factor of 2 yielded the most precise and unbiased results in our simulations. Lastly, an informative normal prior for $log(\kappa)$ was included in the objective function (Eq. T5.6). In earlier versions of the model (results not shown), we found that it was difficult for Length-SRA to determine the difference between a large stock with low productivity and a small stock with high productivity. The inclusion of an informative prior on the κ helps to solve this problem.

2.2. Simulation–evaluation

Model performance was evaluated using a simulation–evaluation based on the biological parameters of a hypothetical fish species. We used the same model structure described in Table 3 for both the simulation and estimation models. However, the operating model was modified to control annual exploitation rate (Eq. T4.2), time-varying selectivity (Eq. T4.4), and observation and process errors.

The simulation model was initialized at unfished conditions (eq. T4.1) but only started reporting data for the simulation–evaluation procedure in year t_{init}. Selectivity in the operating model was computed with the three parameter selectivity function described by Thompson (1994) (Eq. T4.4). We chose to use this three parameter selectivity curve because of its flexibility, which allowed us to switch between logistic and dome-shaped curves in the scenarios in which time-varying selectivity was considered. The observation error in the operating model included lognormal error in the index of abundance and logistic multivariate error (Schnute and Richards, 1995) in the catch numbers-at-length. Recruitment deviations were assumed to be lognormally distributed with constant σ_R (Table 1).

We considered six scenarios in the simulation–evaluation trials, including two selectivity patterns (constant- “C”, and time-varying – “V”) and three historical exploitation rate trajectories (contrast “C”, one-way trip – “O”, and U-ramp – “R”). We use a two-letter acronym to designate the scenarios: CC, CO, CR, VC, VO, and VR. In the constant selectivity scenario, selectivity was assumed to follow a sigmoid shape. In the time-varying selectivity scenario, the selectivity curve was assumed to vary every year, progressively changing from a dome shaped curve to sigmoid and back to dome shaped. In the contrast scenarios the exploitation rate (U_t) starts low and increases beyond U_{target} and then decreases until $U_t \approx U_{target}$. In the one-way trip scenarios U increased through time until $U \approx 2 \cdot U_{target}$. In the U-ramp scenario, U_t increases steadily until $U_t \approx U_{target}$ and remains constant thereafter. Figures showing the U_t and selectivity trajectories are included in the online supplementary materials.

All simulations had 30 years of data and 200 simulation trials were performed for each scenario. We evaluated the distribution of the relative proportional error ($\frac{\text{esimated} - \text{simulated}}{\text{simulated}}$) for the main parameter estimates (R_o, and κ) and for four derived quantities (Depletion: $\frac{\text{SB}_T}{\text{SB}_0}$, $Yield_{target}$, U_{target}, and q). Depletion, $Yield_{target}$ and U_{target} were evaluated for the last year of data only. Additional simulation scenarios, including variability in error levels, misspecification and removal of priors, as well as an additional life history example, are given in the supplementary materials.

2.3. Misspecification of growth parameters – L_∞

One important feature of the Length-SRA is that it assumes that growth follows a von Bertalanffy curve and that the growth parameters are known and constant over time. If this assumption is violated, the model outcomes will be impacted as the model will try to explain the deviations from the true growth curve with changes in the selectivity pattern. Here we illustrate how the outcomes are impacted by the misspecification of L_∞. For the simulations, L_∞ parameter was assumed to be 68 cm. We provided the estimation model with the true value, with 10% overestimation (74.8), and 10% underestimation (61.2). We assumed a simple logistic selectivity curve for this exercise.

2.4. Actual data examples

Two case studies were chosen to illustrate the application of the Length-SRA to actual datasets: Pacific hake and Peruvian jack mackerel. Both species are believed to be subject to time-varying selectivity.

The Pacific hake fishery is believed to exhibit time-varying selectivity due to cohort targeting and annual changes fleet spatial distribution (Ruttan, 2003). The population is known to have spasmodic recruitment, with high recruitment events occurring once or twice every decade (Ressler et al., 2007). Pacific hake tends to segregate by size during their annual migration (Ressler et al., 2007), allowing the fishing fleet to target strong cohorts by changing the spatial distribution of fishing effort as the cohort ages. Hake catch-at-length data were available for the period between 1975 and 2013. The survey index of abundance was available intermittently from 1995 to 2013.

The movement pattern of jack mackerel is not as well known, although fish appear to move between spawning and feeding areas (Gerlotto et al., 2012). Variability in selectivity patterns for the jack mackerel fishery are believed to be associated both with evolution of fleet capacity and gear utilization and with compression and expansion of the species range associated with abundance changes (Gerlotto et al., 2012). Jack mackerel catch at length data was available from 1980 to 2013 and the survey index was available between 1986 and 2013, with the exception of 2010.

We had to make a few assumptions about the population growth parameters and recruitment variability to apply Length-SRA method to these data. For both stocks, stock assessment reports were available. We assumed that the von Bertalanffy parameters were equal to those reported in the assessment documents. We also had to make assumptions regarding the standard deviation of recruitment estimates σ_R. We assumed $\sigma_R = 1.4$ for Pacific hake (Taylor et al., 2014). Estimates of σ_R were not directly reported in the jack mackerel assessment, so we assumed a value of 0.9. We recommend that sensitivity analyses, assuming different values of σ_R, are performed whenever using the Length-SRA for management purposes. Similar assumptions were made for the assumed variability around the survey observations. We assumed $\sigma_{It} = 0.1$ for Pacific hake and $\sigma_{It} = 0.4$ for jack mackerel. We also recommend sensitivity analysis over these parameter values, particularly if the survey estimates are believed to be uncertain.

3. Results

3.1. Simulation–evaluation

We evaluated the performance of the Length-SRA in relation to the main parameters and derived management quantities with boxplots of the relative proportional error. Relative proportional error was calculated based on the maximum likelihood estimates of each parameter for each of the 200 simulated populations for each scenario. Throughout we use the terms positive and negative median bias to indicate that the median relative proportional error is above or below zero. The median relative proportional error sign indicates if a parameter has been underestimated or overestimated the majority of the time.

The simulation–evaluation of Length-SRA showed that R_o estimates showed a small positive median bias for the CC, CO and VO scenarios whereas negative median bias was seen for the CR, VC and VR scenarios. The estimates of κ were either unbised or showed a very small positive median bias. The relative error distribution for κ indicate that the parameter estimates were very precise, likely an effect of the

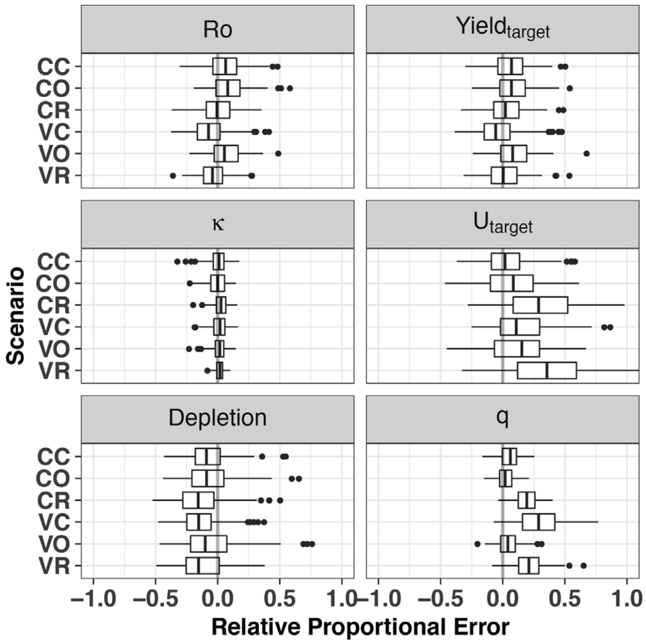

Fig. 1. Relative proportional error for main parameters and derived quantities for all scenarios considered in the simulation–evaluation. Depletion, $Yield_{target}$ and U_{target}) were evaluated for the last year of data. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value ± 1.5· inter-quartile range (distance between the first and third quartiles). CC – constant selectivity and contrast exploitation rate, CO – constant selectivity and one-way trip exploitation rate, CR – constant selectivity and U-ramp exploitation rate, VC- varying selectivity and contrast exploitation rate, VO – varying selectivity and one-way trip exploitation rate, and VR – varying selectivity and U-ramp exploitation rate.

informative prior considered for that parameter, as well as the likelihood function which lets σ_R be higher in the last phase of the estimation (Eq. T5.5) (Fig. 1).

The depletion (SB_t/SB_o) estimates resulted in negative median relative error for all scenarios (Fig. 1). The $Yield_{target}$ relative error distribution was relatively unbiased (median relative error < 9%), with positive median bias occurring for the one-way trip scenarios (CO and VO) and also the CC scenario (Fig. 1). The U_{target} relative error estimates showed positive median bias across all scenarios, with high median bias observed for the CR and VR scenarios. The estimates of q showed positive median bias across all scenarios (Fig. 1).

The simulation–evaluation exercise showed that Length-SRA is able to track the median estimates of selectivity changes through time (Fig. 2). However the selectivity estimates are quite variable, especially for young ages, which is likely due to the observation error in the catch at length composition and to the difficulties in distinguishing strong recruitment events from variability in selectivity.

3.2. Misspecification of growth parameters

Misspecification of L_∞ has severe implications in the capability of Length-SRA to estimate exploitation rate at length, and consequently, selectivity (Fig. 3). The estimates of selectivity were lower than true for young ages and higher than true for older ages if the value of L_∞ was specified to be lower than the true value. In the scenario where L_∞ was set to be higher than the true value, selectivity was estimated to follow a dome shaped pattern. These patterns occur because the model is trying to adjust the mismatch between proportions of catch-at-length and the $P_{l|a}$ matrix by changing the predicted selectivity pattern. As a result, failure to adequately specify L_∞ leads to erroneous estimation of selectivity patterns and, consequently, failure in estimating management quantities.

3.3. Actual data examples

The model fit the Pacific hake and jack mackerel indexes of abundance relatively well (Fig. 4). The Pacific hake index of abundance time series is relatively short and intermittent (survey takes place every two or three years). The index of abundance time series for jack mackerel was longer but it indicates a downward trend in abundance with low contrast in the last ten years of data.

The parameter estimates for the Pacific hake example differ significantly from those in the assessment model. The R_o estimate obtained with Length-SRA was higher, 2.96 billion vs. 2.35 in the Taylor et al. (2014) assessment. The estimate for κ was lower than tha obtained in the assessment, κ = 17.44 from Length-SRA and 25.2 in the Taylor et al. (2014) assessment. For the jack mackerel example, Length-SRA estimated R_o = 3208.29 millions and κ = 15.54. Direct comparisons with the assessment results are not possible because the parameter estimates for the Peruvian jack mackerel stock in isolation are not directly reported in the assessment document (Anonymous, 2013).

The model fit for both species resulted in time-varying selectivities that lead to variation in $Yield_{target}$ and consequent changes in U_{target} (Fig. 4). This is because changes in selectivity result in changes to the vulnerable biomass even if total biomass is constant. A sharp peak in both $Yield_{target}$ and U_{target} is shown for both Pacific hake and jack mackerel. We believe these peaks are likely unrealistic and are associated with difficulties in estimating recruitment deviations in the early years.

The selectivity curves estimated for Pacific hake and jack mackerel are quite variable and mostly estimated to be dome shaped (Fig. 5). The selectivity estimates for Pacific hake are not directly comparable to those given in the 2014 assessment (the year corresponding to the last year of data that we have available; Taylor et al., 2014). Although, several time-varying selectivity scenarios were considered in that assessment document, selectivity at age was generally assumed to be constant after age 6. However, it is possible to see similarities between the overall patterns in selectivity, particularly, a shift towards younger ages in the years of 2010 and 2011 (see Fig. 30 in Taylor et al., 2014). For the jack mackerel example, the selectivity estimates closely match those shown in the 2013 jack mackerel assessment document for the far north fleet, which corresponds to the Peruvian stock used in this example (see Fig. A5.16 in Anonymous, 2013).

The observed variability in selectivity estimates in these examples might indicate real changes in selectivity (e.g. cohort targeting) or might also be caused by misspecification of the growth parameters (see Fig. 3). The growth parameters we used in Length-SRAs were taken from the stock assessment reports. For Pacific hake, however, such estimates are not used in the assessment model as the growth information is overwritten by the empirical weight-at-age data available for that species (Taylor et al., 2014). At this point it impossible to determine the causes for the resulting patterns in selectivity observed with Length-SRA. Further investigation would be needed if this model is to be used for management purposes.

4. Discussion

We present a length-based stock reduction analysis (Length-SRA) that allows monitoring of time-varying selectivity. Catch-at-length is assumed to be known without error in Length-SRA and exploitation rate-at-length is calculated directly from estimates of numbers-at-length. In turn, numbers-at-length are produced based on numbers-at-age and on probabilities derived from growth curve parameters and the assumed variability (standard deviation) around mean length-at-age. This is important because it allows the model to bypass the requirement for the estimation of a selectivity curve, as is required in more

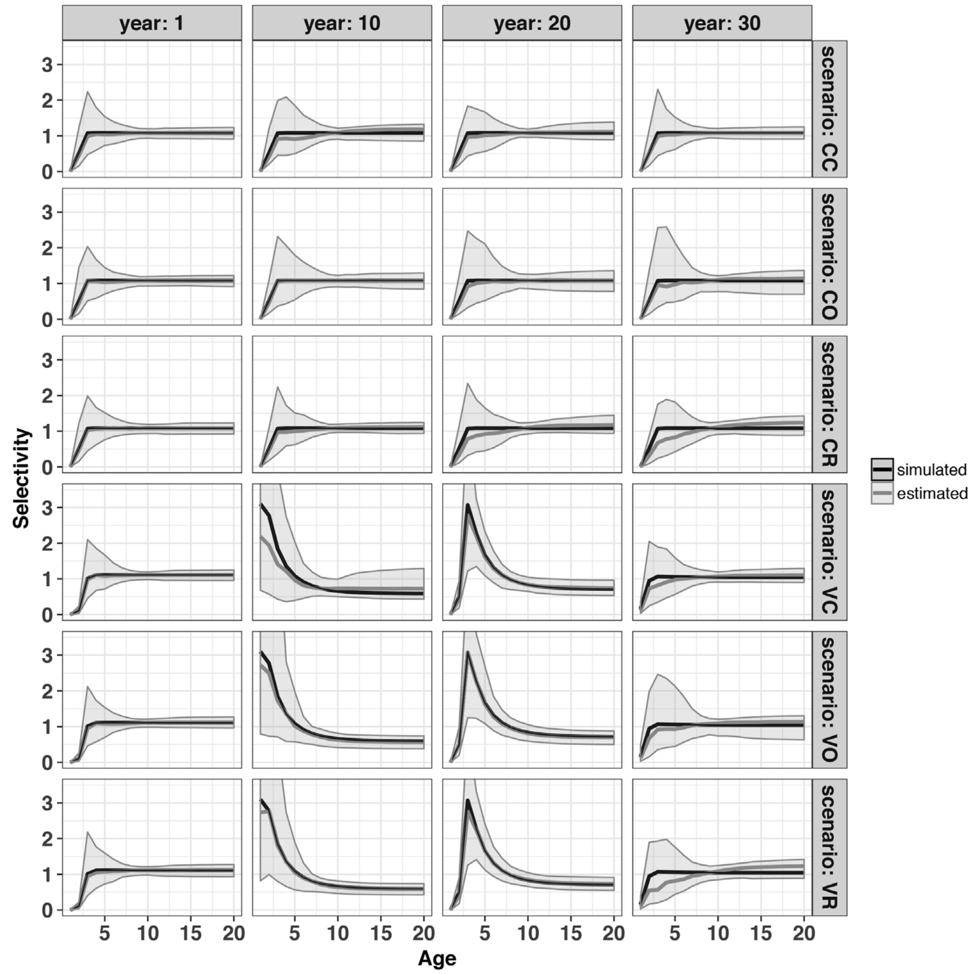

Fig. 2. Simulated and realized selectivity at age estimates for a set of years within simulation–evaluation time series. The estimated solid lines indicate median, 2.5% and 97.5% quantiles for the derived selectivities.

traditional age- and length-based models (e.g. Mesnil and Shepherd, 1990; Sullivan et al., 1990) and in more recent length-based state-space modelling approaches (White et al., 2016). Estimation of selectivity ogives can be very difficult, especially if selectivity is believed to vary over time unpredictably (Linton and Bence, 2011; Martell and Stewart, 2014).

Nielsen and Berg (2014) present a stock assessment approach that accounts for time varying selectivity by treating fishing mortality-at-age as process error estimated within a state-space approach and allowing autocorrelation over ages and time. The accuracy in the estimates of selectivity obtained with Length-SRA are comparable with those presented by Nielsen and Berg (2014), especially for the one-way trip scenarios. However, selectivity estimates from Length-SRA are less precise than those shown by Nielsen and Berg (2014). The high observed variability in selectivity estimates in Length-SRA is probably associated with the assumption of perfect knowledge of the catch-at-length (i.e. no observation error). Nielsen and Berg (2014) explicitly model the observation error in catch-at-age by adding a normally-distributed error term to the catch function. This error is assumed to be zero in Length-SRA. This assumption leads to selectivity estimates that aggregate both the true selectivity effects and the observation error in catch-at-length.

Other recent examples of stock assessment methods that rely on catch-at-length data include Rudd and Thorson (2017) and Hordyk et al. (2015). Rudd and Thorson (2017) presents a length- based stock assessment model, LIME that can account for time-varying recruitment and fishing mortality. LIME, however does not allow for time-varying selectivity. Instead, it assumes that selectivity is constant through time and follows a logistic function. While LIME and the Length-SRA estimates have comparable levels of precision, LIME produces generally more accurate estimates of relative spawner potential ratio. The higher accuracy in LIME is probably associated with the fact that the recruitment deviations in that model are estimated as random effects. This approach drastically reduces the number of estimable parameters, increasing the degrees of freedom and allowing for better performance of the overall model. A future extension of Length-SRA would likely benefit from adopting a similar approach.Hordyk et al. (2015) presents an equilibrium-based model, the LB-SPR, that provides SPR and F/M estimates. LB-SPR produces relatively unbiased estimates of SPR, and has comparable performance to the non-equilibrium method described by Rudd and Thorson (2017), at least for medium and long lived species. In theory, LB-SPR could be applied to each year of data separately to account to changes in selectivity. However, it is important to note that the LB-SPR relies on a parametric assumption regarding the selectivity ogive. Hordyk et al. (2015) show that the results from LB-SPR are quite sensitive to misspecification of the selectivity function.

An important advancement of Length-SRA over conventional stock-assessment models is the indirect calculation of time-varying selectivity. This information alone can be used to characterize the complexity of the fishery system. Length-SRA on its own is reasonably accurate in deriving important management-oriented parameters (depletion and $Yield_{target}$). However another option may be to combine findings from this model with another assessment model, such as a statistical catch-at-age (SCA) model. In this framework, Length-SRA can

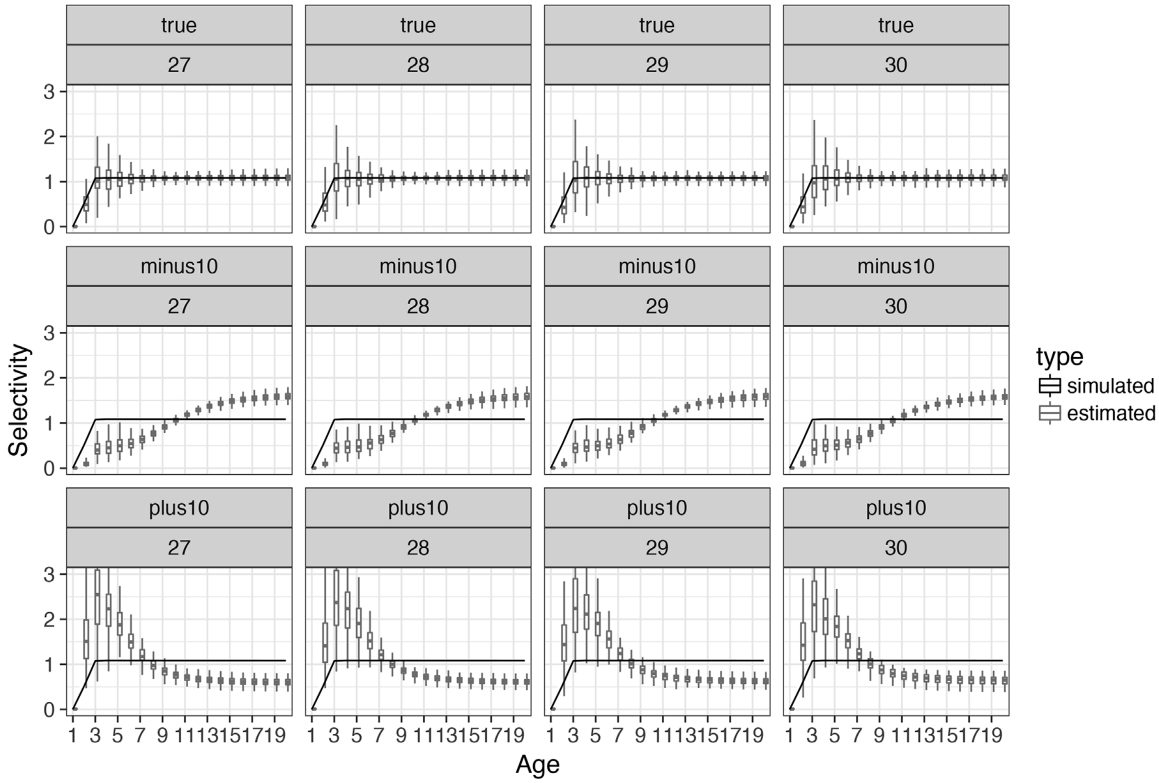

Fig. 3. Simulated and realized selectivity at age when L_∞ is misspecified. Results shown for the last four years of simulation–evaluation time series. Boxplots center lines indicate the median estimate. Lower and upper hinges indicate first and third quartiles. Upper and lower whiskers are given by the maximum and minimum values within the intervals given by the hinge value ± 1.5· inter-quartile range (distance between the first and third quartiles).

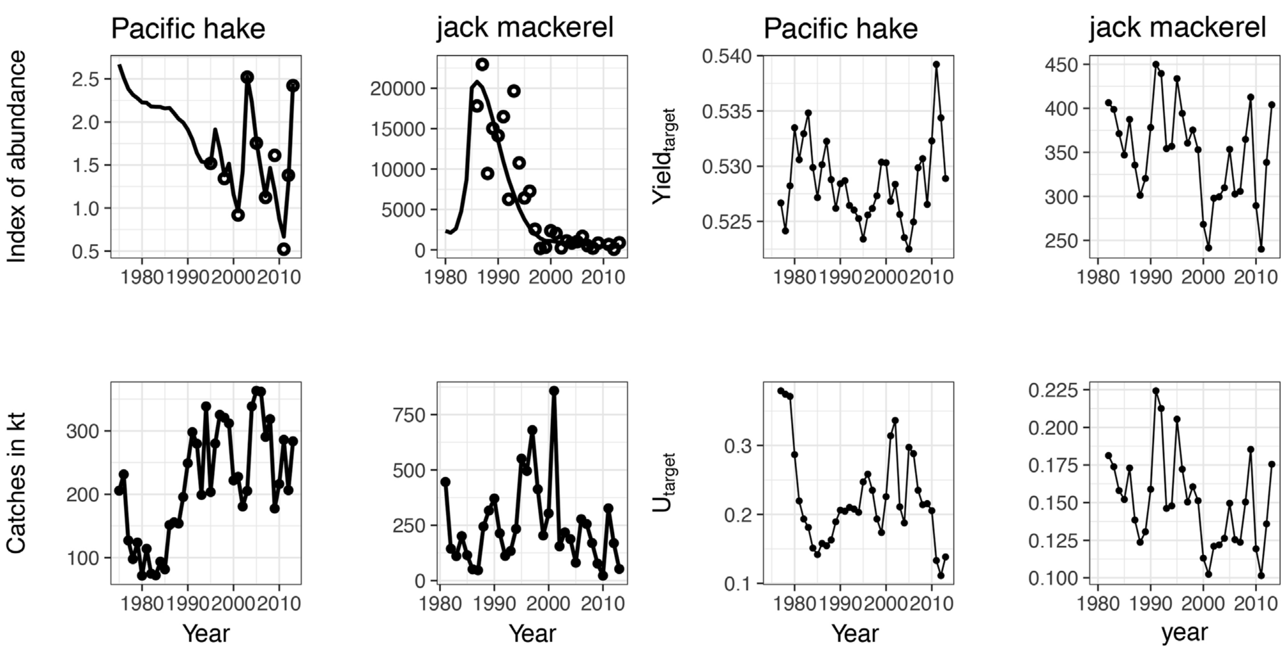

Fig. 4. Fit to index of abundance, historical catches, and $Yield_{target}$ and U_{target} estimates for Pacific hake and jack mackerel. Observed indexes of abundance are shown in open circles, closed dots in $Yield_{target}$ and U_{target} panels indicate model estimates.

be used to calculate annual selectivity patterns and provide an indication of possible changes over time. These selectivity estimates can then become an input into an SCA to calculate other important variables and produce management advice. This combination of models has been used in the past (Walters and Punt, 1994); we suggest Length-SRA may be a useful tool in this context.

Accurate estimates of selectivity are particularly important if fishery management is based on yield-per-recruit reference points. Yield-per-recruit depends on the selectivity curve (Beverton and Holt, 1957) and for this reason, changes in selectivity over time will directly affect reference points (Beverton and Holt, 1957; Hilborn and Walters, 1992). We observed selectivity changes for both Pacific hake and jack

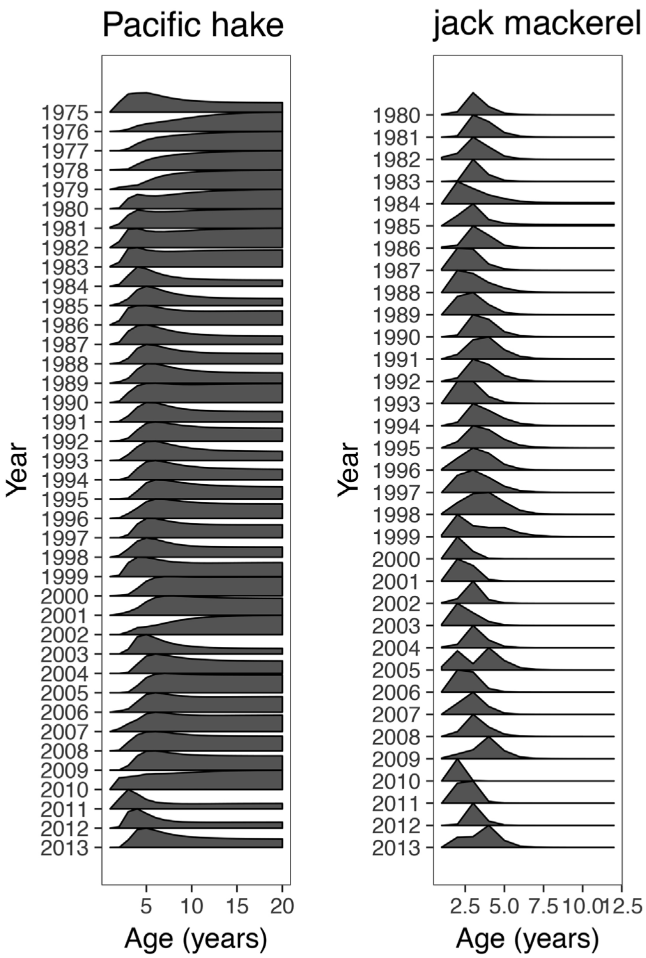

Fig. 5. Estimated selectivity patterns across years for Pacific hake and jack mackerel.

mackerel and show how this variability can lead to a difference between the maximum and minimum estimates of $Yield_{target}$ and U_{target} calculated along the time series. We believe that tracking these changes is important not only to ensure appropriate management recommendations, but also to illustrate the relationship between selectivity patterns and management targets (Vasilakopoulos et al., 2016).

One potential point of concern that should be considered when using Length-SRA is that it assumes that the biological parameters used in the growth curve and catch-at-age relationship are known without error and constant over time. We have tested Length-SRA under misspecification of L_∞ and observed additional bias in the estimates of parameter and management quantities as well as strong distortions in the resulting selectivity parameters. Similarly, Minte-Vera et al. (2017) showed that misspecification in biological parameters, especially in asymptotic length, can have a significant impact in assessment results. Other length models, e.g. MULTIFAN-CL (Fournier et al., 1998), overcome the assumption of known growth parameters by estimating the von Bertalanffy parameters along with the other parameters. The estimation of the growth parameters is made possible by assuming that selectivity follows a parametric function (usually logistic). Once a simple selectivity curve is assumed, all deviations in observed catch-at-length are explained by adjusting the growth parameters. However, this assumption can lead to bias in parameter estimates, as other studies show that variability in selectivity and non-asymptotic patterns are common (Waterhouse et al., 2014). In reality, in most cases it is difficult to know if patterns observed in catch-at-length are caused by fisheries targeting (i.e. selectivity) or if they would be more appropriately explained by adjusting the growth parameters. Therefore, we recommend that, when using Length-SRA, the user should perform extensive sensitivity analyses over the possible range of values for the growth parameters using, for example, the predictive distributions given by the FishLife package (Thorson et al., 2017). Sensitivity analyses are particularly important if the predicted selectivity patterns are highly variable.

The model and simulation exercise presented here assume that the growth parameters are known and constant through time. Consequently, time variation in growth patterns could also impact the results produced by Length-SRA. We would not recommend attempting to estimate time-varying growth parameters within Length-SRA because growth and exploitation rates at length are confounded. However, if estimates of time-varying growth are available, preferably from fishery independent data, those could be used as an input to Length-SRA.

The approach used in Length-SRA is analogous to that used in virtual population analysis in that the length-composition data are assumed to be known without error. For this reason, the selectivity estimates include extra variability due to observation and sampling error. We attempted to minimize this effect by smoothing the predicted selectivity over two years. However this method is not capable of completely removing the observation error effect from the selectivity estimates. Because of the assumption of known catch-at-length, it is important that the catch sampling is representative of the total removals from the population (Pope, 1972). Collecting length-composition information on commercial catch can be very challenging, and the resulting length-composition data can be subject to high variability. Gulland and Rosenberg (1992) cover many examples of how difficulties may arise when sampling length of the catch and provide some advice on how to initiate and design length-composition sampling programs. As in any other fisheries model, biased sampling and/or low sampling effort will result in bias in parameter and fishery reference point estimates (Bunch et al., 2013; Coggins and Quinn, 1998).

Some management parameters are consistently overestimated ($Yield_{target}$) and underestimated (depletion) which may be cause for concern. However, it is important to note that both parameters have low absolute median relative error ($<$ 17%). The magnitude of the bias in the estimates of $Yield_{target}$ and U_{target} observed in this study are comparable (in magnitude) to the results obtained by Martell and Stewart (2014) for MSY and F_{MSY} in a simulation study on the impacts of time-varying selectivity on the estimates generated by a statistical catch-at-age model. Other studies show even higher biases in face of time-varying selectivity (e.g. Henríquez et al., 2016; Linton and Bence, 2011). The estimates of depletion are also comparable to those produced with other SRA-type assessments evaluated by Thorson and Cope (2015). Overall, parameter and derived parameters estimates are generally within the range of many other stock assessment models. As mentioned before, these biases might be reduced by considering a mixed effects approach, with recruitment deviations being considered as random effects, similarly to Rudd and Thorson (2017).

The Length-SRA approach can be a useful tool for fisheries stock assessment. We believe that this is particularly true when time-varying selectivity is thought to occur, especially if the variability is not easily predictable from historical changes in gear use/fleet composition. However, to acknowledge that the selectivity estimates will only be reliable if the growth parameters for the population being assessed are known. In addition, the simple nature of Length-SRA makes it a good candidate model for inclusion on closed-loop simulation studies. Further testing of this model in a closed-loop simulation set up would provide more insight on the model performance on achieving management outcomes (Punt et al., 2016). We foresee the application of this model as an investigative tool to evaluate potential time-varying selectivity patterns, as a stock assessment tool and as part of closed loop simulation studies.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.fishres.2018.07.010.

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